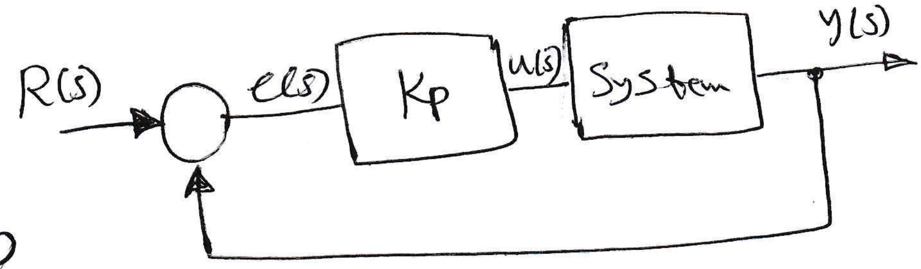
9- Three term Controller (PID) :-

The most basic feedback is a constant "Proportional to error", addition of a term proportional to the Integral of error has a major influence on the system type and steady-state error to polynomials. The final term in the classical structure term proportional to the Derivative of error. Combined, these three terms form the classical PID controller, which widely used in the process and robotics industries.

9-1 Proportional Control (P) :-



$$\frac{U(s)}{E(s)} = KP$$

if we have $G(s) = \dfrac{A}{s^2 + a_1 s + a_2}$, then the characteristic equation is,

$$1 + Kp\,G(s) = 0 \Rightarrow 1 + \frac{A\,Kp}{s^2 + a_1 s + a_2} = 0$$

$$s^2 + a_1 s + a_2 + A K_p = 0$$

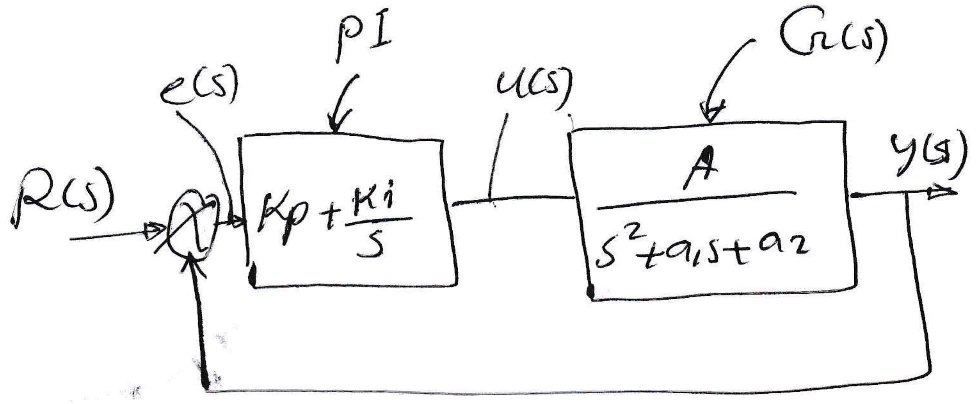
We can control the constant term and the natural frequency. If K_p is made large to get adequate steady-state error, the damping may be much too low for satisfactory transient response.

9-2 Proportional plus Integral Control (PI) :-

$$u(t) = K_p e + K_I \int_{t_0}^{t} e(\tau)\, d\tau.$$

$$\boxed{\frac{U(s)}{E(s)} = D_c(s) = K_p + \frac{K_I}{s}}$$

if we have the plant with $G(s) = \dfrac{A}{s^2 + a_1 s + a_2}$ with a unity feedback,



$$\cancel{T.F. = \frac{Y(s)}{R(s)}} \qquad \frac{U(s)}{E(s)} = \frac{K_p s + K_i}{s}$$

$$T.F. = \frac{Y(s)}{R(s)}$$ or by the characteristic equation

$$1 + D(s)G(s) = 0 \Rightarrow 1 + \frac{K_p s + K_i}{s} * \frac{A}{s^2 + a_1 s + a_2} = 0$$

$$s^3 + a_1 s^2 + a_2 s + K_p s + K_i = 0$$

9-3 Proportional-Integral-Derivative Control (PID).

$$D_c(s) = \frac{U(s)}{E(s)} = K_p + \frac{K_i}{s} + K_d s$$

$$\frac{U(s)}{E(s)} = K_p \left(1 + \frac{K_i}{K_p} \frac{1}{s} + \frac{K_d}{K_p} s\right)$$

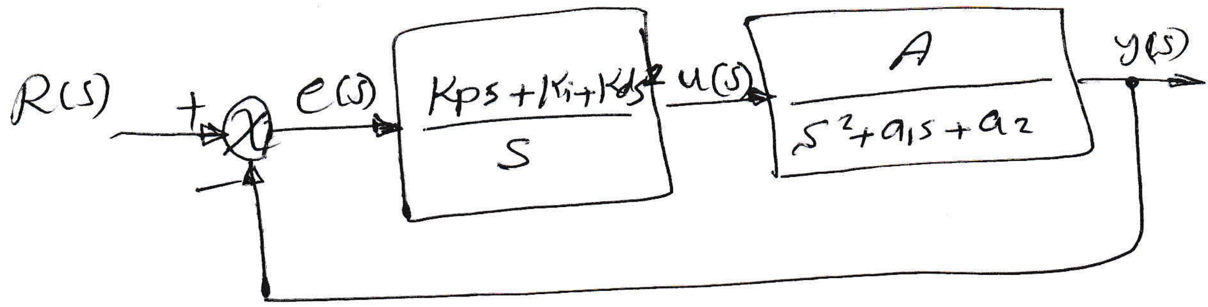
$$\frac{U(s)}{E(s)} = K_p \left(1 + \frac{1}{\frac{K_p}{K_i}} \frac{1}{s} + \frac{K_d}{K_p} s\right)$$

$$\frac{U(s)}{E(s)} = K_p \left(1 + \frac{1}{T_i s} + T_D s\right)$$

$$T_i = \frac{K_p}{K_i}.$$

$$T_D = \frac{K_d}{K_p}.$$

If we have the open loop $G(s) = \frac{A}{s^2+a_1 s+a_2}$ with a unity feedback, we get



the characteristic equation is,

$$1 + D_c(s)\,G(s) = 0$$

$$1 + \frac{K_p s + K_i + K_d s^2}{s} * \frac{A}{s^2+a_1 s+a_2} = 0$$

$$s^3 + a_1 s^2 + a_2 s + (K_p s + K_i + K_d s^2)A = 0$$

$$s^3 + s^2(a_1 + K_d A) + s(a_2 + K_p A) + K_i A = 0$$

Proper choice of K_p, K_D and K_I results in satisfactory transient and steadystate response.
The process of choosing proper K_p, K_d and K_i for a given system is known as "tuning of a PID Controller"

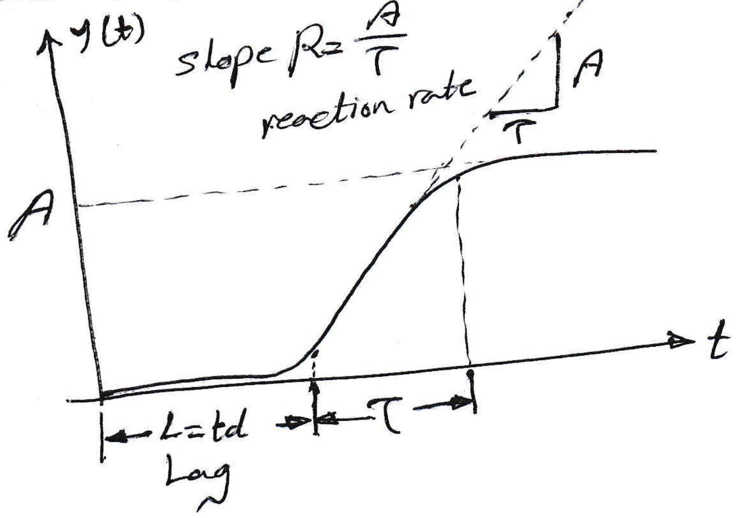
9-4 Ziegler – Nichols Tuning of PID Regulators :-

It is a method for tuning the PID Controller depending on estimates of the plant parameters.

Consider the open loop transfer function as,

$$\frac{Y(s)}{U(s)} = \frac{A e^{-s t_d}}{Ts + 1}$$

which is a first-order system with a time delay of td seconds.

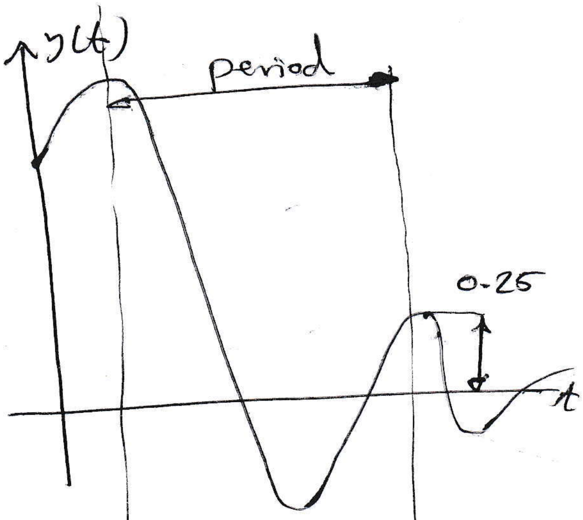


Ziegler and Nichols gave two methods for tuning PID Controller for a such a model. In the first method the choice of Controller parameters is designed to result in a closed-loop step response transient with a decay ratio of approximately 0.25. This means that the transient decays to a quarter of its value after one period of oscillation.

The regulator parameters suggested by Ziegler and Nichols for the controller terms, defined by

$$D_c(s) = K_p\left(1 + \frac{1}{T_I s} + T_D s\right).$$

are given by this table

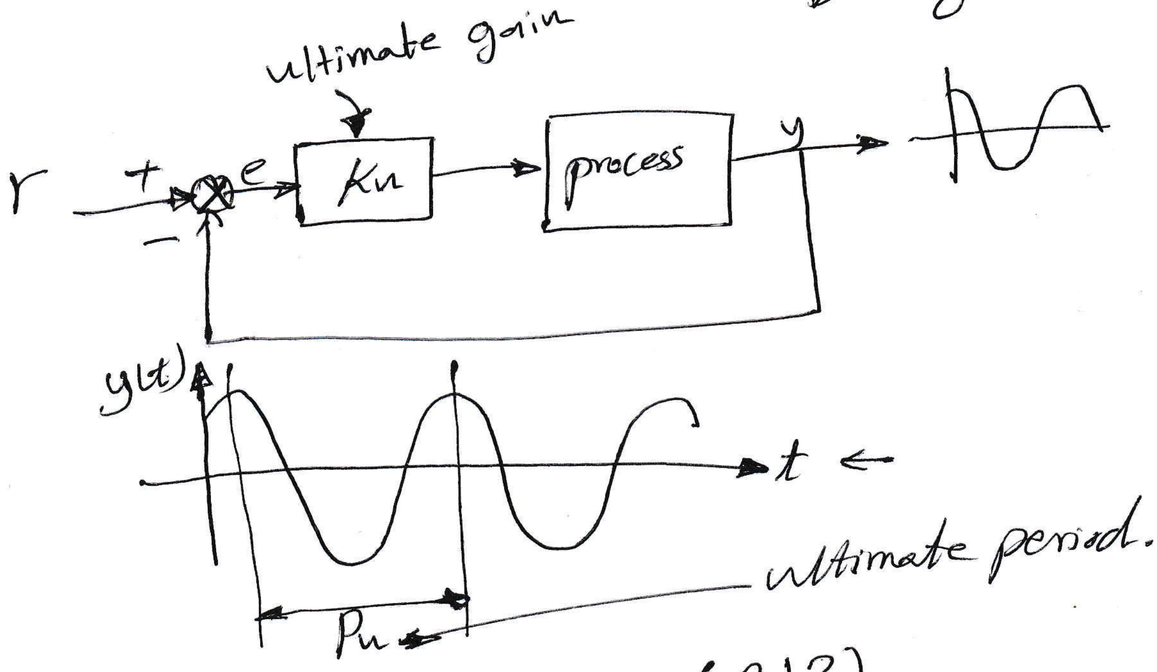


quarter decay ratio

Type of controller	optimum gain
Proportional	$K_p = 1/RL$
PI	$K_p = 0.9/RL$, $T_I = L/0.3$.
PID	$K_p = 1.2/RL$, $T_I = 2L$, $T_D = 0.5L$.

In the ultimate sensitivity method, the criteria for adjusting the parameters are based on evaluating the amplitude and frequency of the oscillations of the system at the limit of stability. the tuning parameters are selected as shown in table below

Type of controller	optimum Gain
Proportional	$K_p = 0.5\,K_u$
PI	$K_p = 0.45\,K_u$, $T_I = \frac{P_u}{1.2}$
PID	$K_p = 0.6\,K_u$, $T_I = \frac{1}{2} P_u$, $T_D = \frac{1}{8} P_u$,



Ex if you have a system with ultimate gain $K_u = 15.3$ and the period is measured as shown in figure below. Determine the proportional and PI regulators according to the Ziegler–Nichols rules based on the ultimate sensitivity method.

Proportional: $K_p = 0.5 K_u$

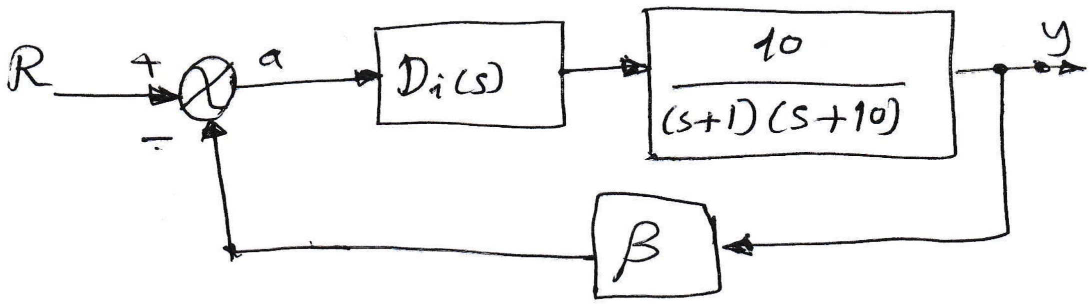
$= 7.65$,

PI : $K_p = 0.45 K_u$

$K_p = 6.885$

$T_I = \frac{1}{1.2} P_u = 35.$

EX ; if you have a system below,



ⓐ if $\beta = 1$,

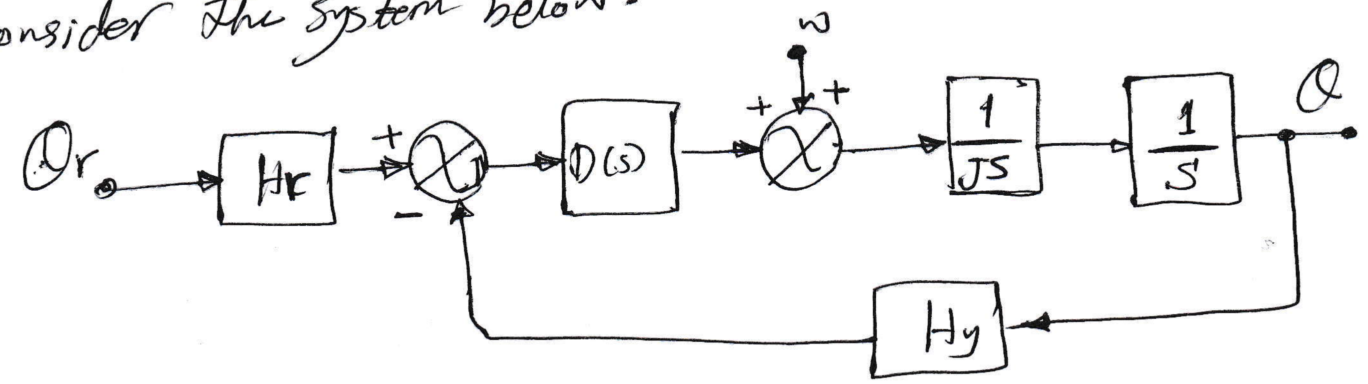
$$D_1(s) = K_p, \quad D_2(s) = \frac{K_p s + K_I}{s}, \quad D_3(s) = \frac{K_p s^2 + K_I s + K_2}{s^2}$$

Choose the controller that will result in a type 1 system with a steady-state error to a unit reference ramp of less than $\frac{1}{10}$.

b) if $\beta = 0.9$, find the steady state error due to a ramp input for your choice of $D_i(s)$ in part (a)

c) If $\beta = 0.9$, what is the system type for part(b).

Q: Consider the system below.



if $J = 10$ Spacecraft inertia, $N\text{-}m\text{-}sec^2/rad$.

θ_r = reference Satellite altitude, rad.

θ = actual Satellite altitude, rad.

$H_y = 1$ Sensor scale factor volts/rad.

$H_r = 1$ reference sensor scale factor, volts/rad.

w = disturbance torque N-m,

(a) Use P, with $D(s) = K_p$ and give the range of values for K_p for which the system will be stable.

(b) Use PI, with $D(s) = (K_p + \frac{K_I}{s})$, and determine the system type and error constant with respect to (i) reference inputs, (ii) disturbance inputs.

c) Use PID, with $D(s) = (K_p + \frac{K_I}{s} + K_d s)$, and determine the system type and error constant with respect to (i) reference input (ii) disturbance input.

(a) $D(s) = K_p$; the characteristic equation is

$$1 + H_y D(s) \frac{1}{Js^2} = 0$$

$$Js^2 + H_y K_p = 0$$

$$\cancel{s = \pm \sqrt{\frac{H_y K_p}{J}}}$$

$$Js^2 = -H_y K_p$$

$$s^2 = -\frac{H_y K_p}{J}$$

$$s = \mp \sqrt{-\frac{H_y K_p}{J}}$$

$s = \mp i \sqrt{\frac{H_y K_p}{J}}$, that means no additional damping is provided. The system cannot be made stable with proportional control alone.

b) (i) the characteristic equation is

$$1 + H_y D(s) \frac{1}{Js^2} = 0$$

with PI control,

$$Js^3 + H_y K_p s + H_y K_I = 0$$

from the Hurwitz's test, the system will always have one pole not in the LHP. Hence, this means it is not a good control strategy.

ii) the same result.

c) (i) the charreteristic equation with PID Control is,

$$1 + H_y \left(K_P + \frac{K_I}{S} + K_D S\right) \frac{1}{J S^2} = 0$$

or $$J S^3 + H_y K_D S^2 + H_y K_P S + H_y K_I = 0$$

$$\frac{\theta(s)}{\theta_r(s)} = H_r \frac{D(s) \frac{1}{J S^2}}{1 + D(s) H_y \frac{1}{J S^2}}$$

$$= \frac{H_r \left(K_P + \frac{K_I}{S} + K_D S\right)}{J S^2 + \left(K_P + \frac{K_I}{S} + K_D S\right) H_y}$$

$$= \frac{H_r (K_D S^2 + K_P S + K_I)}{J S^3 + (K_D S^2 + K_P S + K_I) H_y}$$

if $\theta_r(s) = \frac{1}{S}$, $\theta_{ss} = \frac{H_r}{H_y}$

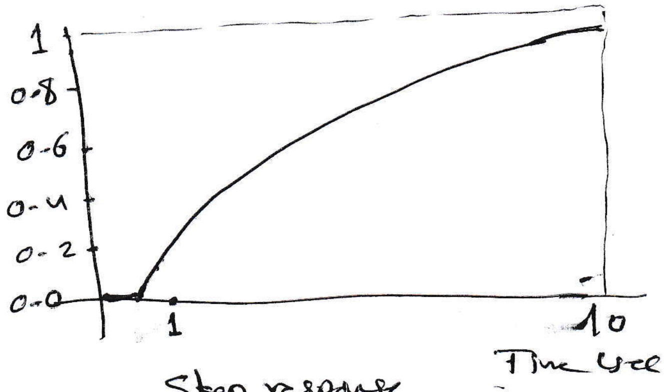
there is zero steady-state error if $H_r = H_y$, the system is type 3 and the error Constant is $K_J = \frac{K_I}{J}$

ii) ~~the~~ $\dfrac{\theta(s)}{W(s)} = \dfrac{s}{Js^3 + H_y(K_D s^2 + K_p s + K_I)}$

If $W(s) = \dfrac{1}{s}$.

$\theta_{ss} = 0$, the system is type 1 and the error constant is $K_v = H_y K_p$.

Q// The Unit-step response of a machine and get the figure below.



Find the proportional P, PI, and PID controller parameters by using the Zeigler–Nichols transient-response method.

from step response : $\cancel{L} = T_d \simeq 0.65$ sec

$$R = \frac{1}{T} \simeq \frac{0.2}{1.25 - 0.65} = 0.33 \ \mathrm{sec}^{-1}$$

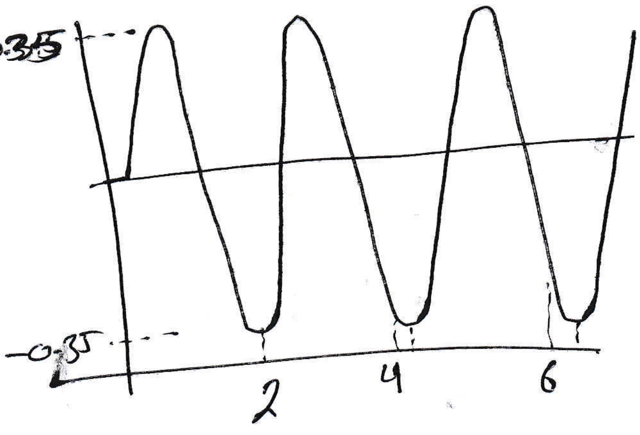
for P $\quad K = \frac{1}{RL} = 4.62$

PI $\quad K = \frac{0.9}{RL} = 4.15 \quad T_I = \frac{L}{0.3} = 2.17$

PID $\quad K = \frac{1.2}{RL} = 5.54 \quad T_I = 2L = 1.3 \quad T_D = 0.5L = 0.33$

PID Control Homework (9)

Q1 if you have the unit impulse response shown in figure below, when the gain $K_u = 8.556$. Determine the proportional P, PI, and PID controller parameters according to the Ziegler – Nichols ultimate sensitivity method.



Q2 if you the transfer function

$$G(s) = \frac{e^{-2s}}{3s+1}$$

a) Find the PID controller parameters using the Zeigler – Nichols tuning rules.

b) the proportional gain of $K_u = 3.044$, as shown by the unit-impulse response. Find the optimal PID controller parameters according to the Ziegler–Nichols tuning rules.